Providing the Supply, Calculating the Round Trip Time for the Most Basic of Cases

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1.0 INTRODUCTION
The first part of this series of articles [1] presented the vertical transportation problem in buildings, which was defined as the design of the elevator system in order to balance the performance of the system against cost.

It concentrated on demand by examining the nature of passenger demand in buildings, looking at the magnitude and random nature of the passenger arrivals.

This article in turn will start examining the operation of the elevator system used to process such passenger demand and provide the adequate supply. It starts by taking a detailed look at the round trip time for the most basic of cases.

2. NOMENCLATURE
\( \tau \) is the round trip time in s
\( int \) is the interval at the main terminal in s
\( L \) is the number of the elevators in the group
\( U \) is the total building population
\( U_i \) is the building population on the \( i^{th} \) floor
\( N \) is the number of floors above the main terminal
\( H \) is the highest reversal floor (where floors are numbered 0, 1, 2….\( N \))
\( S \) is the probable number of stops
\( d_f \) is the typical height of one floor in m
\( v \) is the top rated speed in m/s
\( a \) is the top acceleration in m/s\(^2\)
\( j \) is the top rated speed in m/s\(^3\)
\( t_f \) is the time taken to complete a one floor journey in s assuming that the lift attains the top speed \( v \)
\( P \) is the number of passengers in the car when it leaves the ground (does not need to be an integer)
\( Pr \) is the probability of a certain event taking place
\( Pr_{dest}(i) \) the probability that the destination of a passenger is the \( i^{th} \) floor
\( d_g \) is the height of the ground in m where more than the typical floor height
\( t_{acc} \) is the time taken to accelerate up to the top speed from standstill in s
\( t_{do} \) is the door opening time in s
\( t_{dc} \) is the door closing time in s
\( t_{dec} \) is the time taken to decelerate down from the top speed down to standstill in s
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\( t_{sd} \) is the motor start delay in s
\( t_{ao} \) is the door advance opening time in s (where the door starts opening before the car comes to a complete standstill)
\( t_{pl} \) is the passenger boarding time in s
\( t_{po} \) is the passenger alighting time in s

3.0 INTRODUCING THE ROUND TRIP TIME

The round trip time still forms the basis for the design of elevator traffic systems. Once it has been derived, it can be used to calculate the required number of elevators required in a building in order to meet the traffic design requirements.

A lot of work has been carried out on deriving equations for the round trip time from the most basic of conditions ([2], [3], [4], [5], [6]), to the more complicated cases ([7], [8], [9], [10]), up to the most general of conditions as shown in ([11], [12]). The round trip time has also been evaluated using some modern methods such as Monte Carlo simulation [13] and Markov Chain Monte Carlo (MCMC) [14].

Modern examples of the use of the round trip time in obtaining optimal elevator traffic system designs can be found in [15] and in [16].

This section introduces the definition of the round trip time and some related issues.

3.1 Definition of the Round Trip Time

The round trip time is defined as the time taken for the elevator to pick up the passengers from the main lobby, travel to the upper floors and deliver the passengers to their destinations and then express back to the main terminal again to pick up more passengers. From this point onwards, the round trip time will be denoted with the lowercase Greek letter \( \tau \), pronounced tau.

\( \tau \) must be measured between one event in the cycle and to exactly the same event in the following cycle. For example it could be measured between the doors starting to open at the main terminal until they start re-opening again at the main terminal.

In this article, the round trip time will be derived for the most basic of cases. A simplified diagram showing the layout of a building can be seen in Figure 1. It shows a building with a single entrance floor (previously denoted as an entrance/exit floor in [1]). The type of traffic is incoming only, so all passengers are entering the building from the single entrance floor and heading for the occupant floors. There are \( N \) occupant floors. Each floor has a percentage arrival, proportional to its population. The population of each floor is denoted as \( U_i \). The percentage arrival for each floor can be calculated by dividing \( U_i \) by the total building population \( U \). The probability that a passenger will head for the \( i^{th} \) floor can be calculated as shown below:

\[
Pr_{dest}(i) = \frac{U_i}{U} \quad \text{........(1)}
\]

The movement of the elevator car in the building under incoming traffic condition can be clearly represented using a floor-position vs. time diagram, as shown in Figure 2. The \( y \)-axis represents the position of the car vertically in the building, against the floor height. The \( x \)-axis is the flow of time axis. The diagram shows the car spending some time at the ground floor allowing passengers to board, then moving to the upper floors and allowing passengers to alight at their destination floors, and then the
elevator car expresses back to the ground floor to open its doors and allow more passengers to board the elevator car.

Figure 1: Building overview with a single entrance and incoming traffic only.

Figure 2: General diagram showing a floor-position vs time chart showing the movement of the elevator under single entrance incoming traffic conditions.
The lines showing the movement of the car between the floors has been drawn as a straight line (rather than a curve). This is a simplification and assumes that the acceleration has an infinite value (rather than a finite value).

### 3.2 The assumption under which the equations is derived

The most important conditions under which the round trip time will be derived are:

1. A single entrance only.

2. Incoming traffic only ([1]): Incoming traffic is the traffic in which all passenger journeys originate the entrance/exit floor (in this case there is only one entrance floor) and terminates at an occupant floor (in this case any of the \( N \) occupant floors).

3. Equal floor heights.

4. Top Speed attained in one floor journey: This is shown in more detail in Figure 3, where the rated speed of 1.6 m/s is attained for a period of around 1 second. This is fundamental to the derivation of the round trip time and allows the final expression to be simplified. More details about journey kinematics can be found in [17]. A simple equation used to check whether top speed will be attained in one floor journey is shown in the inequality (2) shown below [17]:

\[
d_{j} \geq \frac{a^{2} \cdot v + v^{2} \cdot j}{a \cdot j}
\]

...........(2)

Figure 3: Journey profile in which top speed is attained.
It is worth noting that when a floor is traversed at top speed, the time taken to traverse the floor can be calculated as shown below:

$$ t_v = \frac{d_f}{v} $$

..........(3)

But when the floor is traversed starting from zero speed and finishing at zero speed, the time taken can be calculated as shown below (assuming that the top speed is attained in this one floor journey) [17]:

$$ t_f = \frac{d_f}{v} + \frac{v}{a} + \frac{a}{j} $$

..........(4)

Thus we can think of the effect of a stop as an extra delay (a form of ‘tax’) equal to:

$$ t_f - t_v = \frac{v}{a} + \frac{a}{j} $$

..........(5)

The highest floor that the elevator reaches in one round trip is called the highest reversal floor, \( H \). It can be less than or equal to \( N \), the number of floor above the main terminal. It does not need to be an integer.

The number of stops that the elevator makes in one round trip is called the probable number of stops, \( S \). It can be less than or equal to the number of passengers in the car, \( P \). It does not need to be an integer.

4.0 THE EXPECTED NUMBER OF STOPS, \( S \)

Fundamental to the derivation of the round trip time for the most basic of cases is the expected number of stops that the elevator will make in the occupant floors. The expected number of stops, \( S \), depends on the number of occupant floors, the population of the occupant floors and the number of passengers boarding the car from the main entrance in each round trip. The derivation of the expected number of stops in a round trip has been based on the following assumptions:

1. Independent passenger destinations (i.e., passenger batch size groups of only one passenger).

2. Incoming traffic only.

The expected number of stops can be derived for both cases of equal occupant floor populations and unequal floor populations. The derivation is independent of whether the top speed is attained one floor journey or not.

The probable number of stops \( S \) is the average number of stops that the elevator makes in one round trip journey. It can be calculated for the case of equal floor populations using equation (6) below:

$$ S = N \left( 1 - \left( \frac{1}{N} \right)^{\rho} \right) $$

..........(6)
Starting from first principles an expression has been derived in this section for the expected number of stops ($S$) that an elevator will make in a round trip journey under incoming traffic conditions only. Under up peak (incoming traffic) conditions, the elevator will fill up with passengers at the ground floor, and then deliver the passengers to their destinations in the upper floors. It then expresses back to the ground floor to collect more passengers and so on.

The car is assumed to fill up with $P$ passengers. The number of floors above ground is denoted by $N$, which is the number of occupant floor above the main entrance.

It shall be assumed that equal floor populations and that passenger destinations are independent (i.e., one passenger’s choice of destination will not influence another passenger’s choice of destination or that passenger arrive in batches of one passenger).

The probability that a passenger will stop at the $i^{th}$ floor:

$$\Pr(a\text{ passenger will stop at floor } i) = \left(\frac{1}{N}\right) \quad \ldots \ldots (7)$$

Where $N$ is the number of floors above the main lobby (or the number of occupant floors). Thus the probability that a passenger will not stop at the $i^{th}$ floor is:

$$\Pr(a\text{ passenger will NOT stop at floor } i) = \left(1 - \frac{1}{N}\right) \quad \ldots \ldots (8)$$

But the car contains $P$ passengers. So the probability that none of them will stop at the $i^{th}$ floor is the product of all of their respective probabilities:

$$\Pr(all\text{ pass will NOT stop at floor } i) = \left(1 - \frac{1}{N}\right)^P \quad \ldots \ldots (9)$$

where $P$ is the number of passengers in the car when it leave the ground floor. The complement of this quantity is the probability that at least one passenger will stop at the $i^{th}$ floor:

$$\Pr(at \text{ least one pass will stop at a floor } i) = 1 - \left(1 - \frac{1}{N}\right)^P \quad \ldots \ldots (10)$$

But this is the same as the probability that a stop will take place on the $i^{th}$ floor. So the probability of stopping on the $i^{th}$ floor is:

$$\Pr(a\text{ stop at floor } i) = 1 - \left(1 - \frac{1}{N}\right)^P \quad \ldots \ldots (11)$$

The expected value of the number of stops can be obtained by adding the probabilities of stopping on all $N$ occupant floors.
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\[ E(\text{number of stops}) = \sum_{i=1}^{N} I \cdot \left(1 - \left(1 - \frac{I}{N}\right)^{P}\right) = N \cdot \left(1 - \left(1 - \frac{I}{N}\right)^{P}\right) \] ...........(12)

Thus the probable number of stops S is equal to:

\[ S = E(\text{number of stops}) = N \cdot \left(1 - \left(1 - \frac{I}{N}\right)^{P}\right) \] ...........(13)

It is worth noting that this expression derived above is still valid in the case of multiple contiguous entrances, where it then represents the expected number of stops, S, that the elevator makes during one round trip in the occupant floors.

It is also worth noting that the formula derived for the expected number of stops is not of much practical value in cases where the floor heights are unequal.

The expected number of stops for the case of unequal floor population can be calculated using equation (14) below. The full derivation can be found in Appendix A.

\[ S = N - \sum_{i=1}^{N} \left(1 - \frac{U_{i}}{U}\right)^{P} \] ...........(14)

5.0 HIGHEST REVERSAL FLOOR, H

An expression for the expected value of the highest reversal floor will be derived for the expected value of the highest reversal floor (H) that an elevator will make in a round trip journey under incoming traffic conditions only.

Under pure incoming traffic conditions, the elevator car will fill up with passengers at the main entrance, and then deliver the passengers to their destinations in the occupant floors. It then expresses back to the main entrance to collect more passengers and so on.

The elevator car is assumed to fill up with P passengers in each round trip. The number of contiguous occupant floors (above the main entrance) is denoted by N. It is assumed that all the occupant floors have equal populations.

The highest reversal floor H is the highest floor that the elevator attains in a round trip journey. It can be evaluated using equation (15) below, for the case of equal floor populations:

\[ H = N - \sum_{i=1}^{N-1} \left(\frac{i}{N}\right)^{P} \] ...........(15)

The derivation of the highest reversal floor for the case of equal floor populations is presented below. The probability that a passenger will stop at the \( i^{th} \) floor:

\[ \Pr(\text{a passenger will head for floor } i) = \left(\frac{1}{N}\right) \] ...........(16)

where N is the number of occupant floors above the main lobby. Thus the probability that a passenger will not head for the \( i^{th} \) floor is:
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Pr(a passenger will NOT head for floor i) = \(1 - \frac{1}{N}\) \(\ldots\) (17)

But the car contains \(P\) passengers. So the probability that none of them will stop at the \(i^{th}\) floor is the product of all of their respective probabilities (assuming independent passengers choices or passenger batch sizes of one):

Pr(all pass will NOT stop at floor i) = \(\left(1 - \frac{1}{N}\right)^P\) \(\ldots\) (18)

The probability that the lift will not travel any higher than a floor \(i\) is the probability that it will not stop on the \((i+1)^{th}\), \((i+2)^{th}\) or \((i+3)^{th}\) \(\ldots\) or the \(N^{th}\) floors. This is expressed as the product of these individual conditional probabilities:

\[
\text{Pr(lift will not travel above floor } i) = \text{Pr}(S_i) \cdot \text{Pr}(S_i \cdot S_{i+1}) \cdot \text{Pr}(S_i \cdot S_{i+1} \cdot S_{i+2}) \ldots \text{Pr}(S_i \cdot S_{i+1} \ldots S_{N-1})
\]
\(\ldots\) (19)

Where \(\text{Pr}(S_i \cdot S_{i+1} \ldots S_{N-1})\) is the probability of not stopping at the \((i+2)^{th}\) floor given that it did not stop at the \(i^{th}\) or \((i+1)^{th}\) floors.

\[
\text{Pr(lift will not travel above floor } i) = \left(1 - \frac{1}{N}\right)^{P} \cdot \left(1 - \frac{1}{N-1}\right)^{P} \cdot \left(1 - \frac{1}{N-2}\right)^{P} \ldots \left(1 - \frac{1}{i+2}\right)^{P} \cdot \left(1 - \frac{1}{i+1}\right)^{P}
\]
\(\ldots\) (20)

It is worth nothing that the reason for \((i+1)^{th}\) term in the last part is due to the fact that there are only \(i+1\) choices remaining to choose from when looking at the probability of not stopping at floor \(N\) \((i\) floors and 1 floor, which is floor \(N\) itself).

This can be re-written as:

\[
\text{Pr(lift will not travel above floor } i) = \left(\frac{N-1}{N}\right)^{P} \cdot \left(\frac{N-2}{N-1}\right)^{P} \cdot \left(\frac{N-3}{N-2}\right)^{P} \ldots \left(\frac{i+1}{i+2}\right)^{P} \cdot \left(\frac{i}{i+1}\right)^{P}
\]
\(\ldots\) (21)

Putting all terms inside the same bracket gives:

\[
\text{Pr(lift will not travel above floor } i) = \left(\left(\frac{N-1}{N}\right) \cdot \left(\frac{N-2}{N-1}\right) \cdot \left(\frac{N-3}{N-2}\right) \ldots \left(\frac{i+1}{i+2}\right) \cdot \left(\frac{i}{i+1}\right)\right)^{P}
\]
\(\ldots\) (22)

This simplifies to:

\[
\text{Pr(lift will not travel above floor } i) = \left(\frac{i}{N}\right)^{P}
\]
\(\ldots\) (23)
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Note that this is also equal to the probability that the highest reversal floor will be any of the floors 1 to \(i\). In other words, it is the probability that the highest reversal floor will be floor 1 or 2 or 3…..or \(i\), which is equal to the sum of these probabilities.

\[
\Pr(\text{lift will not travel above floor } i) = \left( P(H = 1) + P(H = 2) + P(H = 3) + \ldots + P(H = i) \right) \tag{24}
\]

Applying the same argument to the \((i-1)\)th floor, gives the following:

\[
\Pr(\text{lift will not travel above floor } i-1) = \left( \frac{i-1}{N} \right)^p \tag{25}
\]

It is worth noting that this is also equal to the probability that the highest reversal floor will be any of the floors 1 to \(i-1\). In other words, it is the probability that the highest reversal floor will be floor 1 or 2 or 3…..or \(i-1\), which is equal to the sum of these probabilities.

\[
P(\text{lift will not travel above floor } i) = \left( P(H = 1) + P(H = 2) + P(H = 3) + \ldots + P(H = i-1) \right) \tag{26}
\]

If we subtract the two expressions from each other (i.e., expression (26) from (24)) only one term remains (which is the probability that the highest reversal floor is the \(i\)th floor). So the probability that the \(i\)th floor is the highest reversal floor is the probability that the elevator does not travel above the \(i\)th floor minus the probability that the elevator does not travel above the \((i-1)\)th floor. Thus:

\[
\Pr(H = i) = \left( \frac{i}{N} \right)^p - \left( \frac{i-1}{N} \right)^p \tag{27}
\]

But any of the floors from 1 to \(N\) could be the highest reversal floor. The expected value of the highest reversal is thus the weighted average of all the possible highest reversal floors (i.e., 1 to \(N\)):

\[
\text{expected value(highest reversal floor)} = E(H) = \sum_{i=1}^{N} i \cdot \left( \frac{i}{N} \right)^p - \left( \frac{i-1}{N} \right)^p \tag{28}
\]

Re-arranging the two terms will make it easier to simplify later on:

\[
E(H) = \sum_{i=1}^{N} i \cdot \left( \frac{i}{N} \right)^p - \left( \frac{i-1}{N} \right)^p + \left( \frac{i}{N} \right)^p \tag{29}
\]

Expanding gives:
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\[
E(H) = 1 \left( \left( \frac{1}{N} \right)^p + \left( \frac{1}{N} \right)^p \right) + 2 \left( \left( \frac{2}{N} \right)^p + \left( \frac{2}{N} \right)^p \right) + 3 \left( \left( \frac{3}{N} \right)^p + \left( \frac{3}{N} \right)^p \right) + \ldots \\
(N-1) \left[ \left( \frac{N-2}{N} \right)^p + \left( \frac{N-1}{N} \right)^p \right] + (N) \left[ \left( \frac{N-1}{N} \right)^p + \left( \frac{N}{N} \right)^p \right] = \ldots \ldots \ldots (30)
\]

Simplifying gives:

\[
E(H) = 0 - \left( \frac{1}{N} \right)^p - \left( \frac{2}{N} \right)^p - \left( \frac{3}{N} \right)^p \ldots \ldots - \left( \frac{N-1}{N} \right)^p + N \ldots \ldots (31)
\]

This can be rearranged as:

\[
E(H) = N - \sum_{i=1}^{N-1} \left( \frac{i}{N} \right)^p \ldots \ldots (32)
\]

Note that the summation extends only to \( N-1 \) (i.e., not \( N \)). It is worth noting that the maximum possible value of \( H \) is \( N \), which is to be expected at high values of \( P \). In the case of multiple contiguous entrances, the equation above can still be used and it then represents the expected value of the highest reversal floor in the occupant floors. A corresponding value, denoted as \( L \) could be found for the lowest reversal floor in the contiguous entrance floors.

The expected value of the highest reversal for the case of unequal floor population can be calculated using equation (33) below. The full derivation can be found in Appendix B.

\[
H = N - \sum_{j=1}^{N-1} \left( \sum_{i=1}^{j} \frac{U_i}{U} \right)^p \ldots \ldots (33)
\]

Where:
\( N \) is the number of occupant floors
\( H \) is the expected value of the highest reversal floor (in unit of floors)
\( U_i \) is the population of the \( i \)th floor
\( U \) is the total building population

6.0 DERIVATION OF THE EQUATION FOR THE ROUND TRIP TIME UNDER THE MOST BASIC OF CASES

In this section, the equation for the round trip time equation is derived from first principles. It makes the following assumption:
1. The traffic is pure incoming traffic (up peak only). All passengers arrive at the entrance and board the elevator to go to his/her destination on one of the upper floors.

2. In one trip time, the elevator makes $S$ stops and reaches $H$ highest reversal floor. These two variables have been derived elsewhere. They depend on the number of occupant floors above the main entrance (the lobby), $N$, and on the number of passengers in the car, $P$. They can be derived for the general case where the floor populations are unequal or for the special case where the floor populations are equal.

3. The elevator collects $P$ passengers from the main terminal (lobby) and delivers them to their selected destinations. It then expresses back to the main terminal to pick another set of $P$ passengers.

4. Every time the elevators stops at a floor, it takes time for the doors to open, the passengers to transfer (in at the main terminal and out at the destination floors) and then for the doors to close. Further delay at each stop is caused by the start delay of the motor. The delay will be reduced in the case of advanced door opening.

5. The assumption will be made that the elevator will attain its top speed in one floor journey.

6. The assumption will also be made that the floor heights are equal.

7. The assumption will also be made that there is only one entrance floor (arrival floor). All passengers arrive through this single entrance.

The round trip time is made up of main three parts: the time spent at the ground floor collecting passengers, the time spent travelling to the upper floors and delivering passengers to their destination, and the third part is the express travel back from the highest reversal back to the main terminal.

The time spent at the main terminal involves the opening of the door, the transfer of $P$ passengers into the car and the closing of the door, in addition to the start delay minus the advanced door opening.

\[
\tau = \tau_{MT} + \tau_{S} + \tau_{H}
\] ..........(34)

Where:

$\tau_{MT}$ is the time spent at the main terminal in s

$\tau_{S}$ is the time spent travelling to the upper destination floors and delivering the passengers in s

$\tau_{H}$ is the time spent expressing back to the main terminal from the highest reversal floor in s

The time component spent at the main terminal can be found as shown in equation (35) below:
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\[ \tau_{MT} = (t_{do} + t_{dc} + t_{sd} - t_{ao}) + P(t_{pi}) \]  \hspace{1cm} \text{(35)}

The stopping time can be calculated as shown in equation (36) below:

\[ \tau_s = \left(S \cdot (t_{acc} + t_{dec}) + \left(\frac{H \cdot d_f}{v}\right)\right) + S \cdot (t_{do} + t_{dc} + t_{sd} - t_{ao}) + P(t_{po}) \]

\[ = H \cdot \left(\frac{d_f}{v}\right) + S \cdot (t_{acc} + t_{dec} + t_{do} + t_{dc} + t_{sd} - t_{ao}) + P(t_{po}) \]  \hspace{1cm} \text{(36)}

But the time to traverse on floor height assuming that top speed is attained in one floor journey is:

\[ t_f = \frac{d_f}{v} + t_{acc} + t_{dec} = t_v + t_{acc} + t_{dec} = \frac{d_f}{v} + \frac{v}{a} + \frac{a}{j} \]  \hspace{1cm} \text{(37)}

Thus the stopping time can be expressed as shown below:

\[ \tau_s = H \cdot \left(\frac{d_f}{v}\right) + S \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P(t_{po}) \]  \hspace{1cm} \text{(38)}

The third part of the round trip time equation is the time taken to travel back from the highest reversal floor express back to the main terminal:

\[ \tau_H = \left(\frac{H \cdot d_f}{v} + \frac{v}{a} + \frac{a}{j}\right) = \left(\frac{H \cdot d_f}{v}\right) + \left(\frac{v}{a} + \frac{a}{j}\right) = \left(\frac{H \cdot d_f}{v}\right) + \left(t_f - \frac{d_f}{v}\right) \]  \hspace{1cm} \text{(39)}

Adding all the three elements together gives:

\[ \tau = \tau_{MT} + \tau_s + \tau_H \]

\[ = (t_{do} + t_{dc} + t_{sd} - t_{ao}) + P(t_{pi}) + H \cdot \left(\frac{d_f}{v}\right) + S \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P(t_{po}) \]

\[ + \left(\frac{H \cdot d_f}{v}\right) + \left(t_f - \frac{d_f}{v}\right) \]  \hspace{1cm} \text{(40)}

\[ = 2 \cdot H \cdot \left(\frac{d_f}{v}\right) + (S + 1) \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P(t_{pi} + t_{po}) \]

The equation for the round trip time can be written as follows:

\[ \tau = 2 \cdot H \cdot \left(\frac{d_f}{v}\right) + (S + 1) \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P(t_{pi} + t_{po}) \]  \hspace{1cm} \text{(41)}
Equation (41) above assumes equal floor heights. In cases where the height of the ground floor is more than other floor, the equation can be amended to account for that:

\[
\tau = 2 \cdot H \cdot \left( \frac{d_f}{v} \right) + (S + 1) \cdot \left( t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{al} - t_{ao} \right) + P(t_{pi} + t_{po}) + 2 \cdot \left( \frac{d_G - d_f}{v} \right) \quad \text{(42)}
\]

It is worth noting that the last term has been added for the case where the main terminal (the lobby) has a floor height that is more than the typical floor height. As this distance is covered in both the up and down directions, it has been multiplied by 2. \(d_G\) is the height of the main terminal floor.

In general for most buildings (especially office buildings) the main terminal or the lobby has a height greater than the typical floor height, usually for aesthetic reasons but sometimes for functional reasons as well.

### 7.0 ASSUMPTIONS IN DERIVING THE BASIC ROUND TRIP TIME EQUATION

When deriving the basic equation for the round trip time, the following assumptions have been made:

1. Passenger choices of floors are independent of each other (this affects the derivation of \(H\) and \(S\)).

2. Plentiful supply of passenger waiting at the main entrance. It has been assumed that there are always sufficient numbers of passengers waiting in the lobby to board the elevator car, resulting in no extra delays to the operation of the elevator car. In reality passengers arrive randomly in a process that is best represented by a Poisson arrival process.

3. An important assumption made in the derivation of the round trip time equation is that the top speed is attained in one floor journey. This is not correct in many cases where the speed is above 2.5 m·s⁻¹.

4. It has been assumed that only one passenger is boarding or alighting at the same time.

5. The only type of traffic present is the incoming traffic (up peak traffic).

6. Equal floor heights have been assumed.

7. It has been assumed that the doors starts closing immediately after the last passenger has boarded or alighted. In reality there will be delay depending on the timer controlling the door operation, and depending on whether other passengers use the door close button.

8. It has been assumed that passengers enter the building from one single entrance. In reality many buildings have underground car parks or different level street entrances.
9. No door re-openings have been assumed.

10. It has been assumed that all lifts in the same group serve all floors and that any passenger regardless of his/her destination can board any available lift.

8.0 THE CASE OF BATCH ARRIVALS

Historically, the development of the round trip time equations has assumed independent passenger decisions. In other words, it assumes that passengers are making their destination decisions independently of each other. This can also be expressed as saying that passenger arrive in batches that have a size of one passenger. This has an effect when calculating the probability none of the passengers stopping at a floor, as it is equal to the probability of any one passenger not going to a floor, raised to the power P, assuming that the decisions of the P passengers are independent events.

When passengers are travelling in groups (i.e., more than one passenger in the group) this independence is no longer valid, and the equations are modified to reflect this by dividing the number of passengers by the average batch size [18].

REFERENCES
Appendix A: Derivation of the formula for the expected number of stops under unequal floor populations

It shall be assumed that equal floor populations and that passenger destinations are independent (i.e., one passenger’s choice of destination will not influence another passenger’s choice of destination or that passenger arrive in batches of one passenger).

The probability that a passenger will stop at the \(i\)th floor:

\[
Pr(\text{a passenger will stop at floor } i) = \left( \frac{U_i}{U} \right)
\]  .........(43)

Where \(U_i\) is the population of the \(i\)th floor and \(U\) is the total building population. Thus the probability that a passenger will not stop at the \(i\)th floor is:

\[
Pr(\text{a passenger will NOT stop at floor } i) = \left( 1 - \frac{U_i}{U} \right)
\]  .........(44)

But the car contains \(P\) passengers. So the probability that none of them will stop at the \(i\)th floor is the product of all of their respective probabilities (assuming independent passenger decisions or batch sizes of only one passenger):
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Pr(all pass will NOT stop at floor i) = \left(1 - \frac{U_i}{U}\right)^P 

\text{.........(45)}

where P is the number of passengers in the car when it leaves the ground floor. The complement of this quantity is the probability that at least one passenger will stop at the $i^{th}$ floor:

Pr(at least one pass will stop at a floor i) = 1 - \left(1 - \frac{U_i}{U}\right)^P 

\text{.........(46)}

But this is the same as the probability that a stop will take place on the $i^{th}$ floor. So the probability of stopping on at the $i^{th}$ floor is:

Pr(a stop at floor i) = 1 - \left(1 - \frac{U_i}{U}\right)^P 

\text{.........(47)}

The expected value of the number of stops can be obtained by adding the probabilities of stopping on all $N$ occupant floors.

\[ E(\text{number of stops}) = \sum_{i=1}^{N} 1 \cdot \left(1 - \left(1 - \frac{U_i}{U}\right)^P\right) = N \sum_{i=1}^{N} \left(1 - \frac{U_i}{U}\right)^P \] 

\text{.........(48)}

Thus the probable number of stops $S$ for the case of unequal floor populations is equal to:

\[ S = E(\text{number of stops}) = N - \sum_{i=1}^{N} \left(1 - \frac{U_i}{U}\right)^P \] 

\text{.........(49)}

It is worth noting that this expression derived above is still valid in the case of multiple contiguous entrances, where it then represents the expected number of stops, $S$, that the elevator makes during one round trip in the occupant floors.

Appendix B: Derivation of the formula for the highest reversal floor assuming unequal floor populations

The expected value of the highest reversal floor under unequal floor populations is derived below. The population of the $i^{th}$ occupant floor will be denoted as $U_i$.

The probability of the elevator not stopping at the $i^{th}$ floor will be first derived. Assuming unequal floor populations the probability that a passenger will stop at the $i^{th}$ floor:

\[ \Pr(\text{a passenger will NOT head for floor i}) = \left(1 - \frac{U_i}{U}\right)^P \] 

\text{.........(50)}
But the car contains $P$ passengers. So the probability that none of them will stop at the $i^{th}$ floor is the product of all of their respective probabilities (assuming independent passengers choices or passenger batch sizes of one) which is also the probability of the elevator not stopping at the $i^{th}$ floor:

$$\Pr(\text{all pass will NOT stop at floor } i) = \left(1 - \frac{U_i}{U}\right)^p \quad \ldots \ldots (51)$$

The probability that the lift will not travel any higher than a floor $i$ is the probability that it will not stop on the $(i+1)^{th}$, $(i+2)^{th}$ or $i+3^{th}$........ or the $N^{th}$ floors. This is expressed as the product of these individual probabilities (where all of these probabilities are conditional probabilities):

$$\Pr(\text{lift will not travel above floor } i) = \Pr(S_i') \cdot \Pr\left(S_{i+1}'S_i\right) \cdot \Pr\left(S_{i+2}'S_{i+1}\right) \ldots \ldots \Pr\left(S_N'S_{i+1}\ldots S_{N-1}\right) \quad \ldots \ldots (52)$$

Where $\Pr(S_{i+1}'S_i)$ is the probability of not stopping at the $(i+2)^{th}$ floor given that it will not stop at the $i^{th}$ or $(i+1)^{th}$ floors.

$$\Pr(\text{lift will not travel above floor } i) = \left(1 - \frac{U_{i+1}}{U}\right)^p \cdot \left(1 - \frac{U_{i+2}}{U - U_{i+1}}\right)^p \cdot \left(1 - \frac{U_{i+3}}{U - U_{i+1} - U_{i+2}}\right)^p \ldots \ldots (53)$$

It is worth nothing that the reason for $(i+1)^{th}$ term in the last part is due to the fact that there are only $i+1$ choices remaining to choose from when looking at the probability of not stopping at floor $N$ ($i$ floors and 1 floor, which is floor $N$ itself).

This can be re-written as:

$$\Pr(\text{lift will not travel above floor } i) = \frac{U - U_{i+1}}{U} \cdot \frac{U - U_{i+1} - U_{i+2}}{U - U_{i+1}} \cdot \frac{U - U_{i+1} - U_{i+2} - U_{i+3}}{U - U_{i+1} - U_{i+2}} \ldots \ldots (54)$$

Simplifying by cancelling identical terms in the numerators and denominators of consecutive brackets gives:

$$\Pr(\text{lift will not travel above floor } i) = \left(\frac{U - U_{i+1} - U_{i+2} \ldots - U_{N-1} - U_N}{U}\right)^p \quad \ldots \ldots (55)$$
This can be simplified to the following compact form:

\[
\Pr(\text{lift will not travel above floor } i) = \left( \frac{U_1 + U_2 + U_3 + \ldots + U_i}{U} \right)^p = \left( \frac{\sum_{k=1}^{i} U_k}{U} \right)^p \quad \text{........(56)}
\]

It is worth noting that this is also equal to the probability that the highest reversal floor will be any of the floors 1 to \( i \). In other words, it is the probability that the highest reversal floor will be floor 1 or 2 or 3…..or \( i \), which is equal to the sum of these probabilities.

\[
\Pr(\text{lift will not travel above floor } i) = 
\left( P(H = 1) + P(H = 2) + P(H = 3) + \ldots + P(H = i) \right) 
\quad \text{........(57)}
\]

Using the result above, we can also find the probability of the elevator not travelling above the \((i-1)\)th floor, by applying the same argument to the \((i-1)\)th floor, gives the following:

\[
\Pr(\text{lift will not travel above floor } i - 1) = \left( \frac{\sum_{k=1}^{i-1} U_k}{U} \right)^p \quad \text{........(58)}
\]

It is worth noting that this is also equal to the probability that the highest reversal floor will be any of the floors 1 to \( i-1 \). In other words, it is the probability that the highest reversal floor will be floor 1 or 2 or 3…..or \( i-1 \), which is equal to the sum of these probabilities.

\[
P(\text{lift will not travel above floor } i) = 
\left( P(H = 1) + P(H = 2) + P(H = 3) + \ldots + P(H = i - 1) \right) 
\quad \text{........(59)}
\]

If we subtract the two expressions from each other (i.e., (59) from (57)) only one term remains (which is the probability that the highest reversal floor is the \( i \)th floor). So the probability that the \( i \)th floor is the highest reversal floor is the probability that the lift does not travel above the \( i \)th floor minus the probability that the lift does not travel above the \((i-1)\)th floor. Thus:

\[
\Pr(\text{floor } i \text{ is the highest reversal floor}) = \left( \frac{\sum_{k=1}^{i} U_k}{U} \right)^p - \left( \frac{\sum_{k=1}^{i-1} U_k}{U} \right)^p 
\quad \text{........(60)}
\]

But any of the floors from 1 to \( N \) could be the highest reversal floor. The expected value of the highest reversal is the weighted average of all the possible highest reversal floors (i.e., 1 to \( N \)).
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\[ E(H) = \sum_{i=1}^{N} (i \cdot \Pr(H = i)) = \sum_{i=1}^{N} \left( i \cdot \left( \left( \sum_{k=i+1}^{N} U_k \right)^p \right) - \left( \sum_{k=i+1}^{N} \frac{U_k}{U} \right)^p \right) \]  

..........(61)

Re-arranging the two terms will make it easier to simplify later on:

\[ E(H) = \sum_{i=1}^{N} \left( i \cdot \left( \left( \sum_{k=i+1}^{N-1} U_k \right)^p \right) + \left( \sum_{k=i+1}^{N} \frac{U_k}{U} \right)^p \right) \]  

..........(62)

Expanding gives:

\[ E(H) = 1 \cdot \left( -\left( \frac{0}{U} \right)^p + \left( \frac{U_1}{U} \right)^p \right) + 2 \cdot \left( -\left( \frac{U_1}{U} \right)^p + \left( \frac{U_1+U_2}{U} \right)^p \right) + 3 \cdot \left( -\left( \frac{U_1+U_2}{U} \right)^p + \left( \frac{U_1+U_2+U_3}{U} \right)^p \right) + \ldots \]  

\[ (N-1) \cdot \left( -\left( \frac{U_1+U_2+\ldots+U_{N-2}}{U} \right)^p + \left( \frac{U_1+U_2+\ldots+U_{N-2}+U_{N-1}}{U} \right)^p \right) + \]  

\[ (N) \cdot \left( -\left( \frac{U_1+U_2+\ldots+U_{N-2}+U_{N-1}}{U} \right)^p + \left( \frac{U_1+U_2+\ldots+U_{N-2}+U_{N-1}}{U} \right)^p \right) + \]  

..........(63)

Rearranging gives:
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\[ E(H) = \left( -\left( \frac{0}{U} \right)^{p} + \left( \frac{U_1}{U} \right)^{p} \right) + \left( -2 \cdot \left( \frac{U_1}{U} \right)^{p} + 2 \cdot \left( \frac{U_1 + U_2}{U} \right)^{p} \right) + \right. \\
\left. \left( -3 \cdot \left( \frac{U_1 + U_2}{U} \right)^{p} + 3 \cdot \left( \frac{U_1 + U_2 + U_3}{U} \right)^{p} \right) \right) \\
\left( -N - 1 \cdot \left( \frac{U_1 + U_2 + U_3 + \ldots + U_{N-2} + U_{N-1}}{U} \right)^{p} \right) + \\
\left( N - 1 \cdot \left( \frac{U_1 + U_2 + U_3 + \ldots + U_{N} + U_{N+1}}{U} \right)^{p} \right) \\
\left( N \cdot \left( \frac{U_1 + U_2 + U_3 + \ldots + U_{N-2} + U_{N-1}}{U} \right)^{p} \right)
\]

\[ \text{Simplifying gives:} \]

\[ E(H) = 0 - \left( \frac{U_1}{U} \right)^{p} - \left( \frac{U_1 + U_2}{U} \right)^{p} - \left( \frac{U_1 + U_2 + U_3}{U} \right)^{p} \ldots. \]

\[ \ldots. + \left( \frac{U_1 + U_2 + U_3 + \ldots + U_{N-1}}{U} \right)^{p} + N \]

\[ \text{This can be rearranged as:} \]

\[ E(H) = N - \sum_{i=1}^{N-1} \left( \left( \sum_{j=1}^{i} \left( \frac{U_j}{U} \right) \right)^{p} \right) \]

\[ \ldots. \]

\[ \text{It is worth noting that this result is also valid for the case of contiguous multiple entrances, where it represents the highest reversal floor in the occupant floors.} \]