Modern Elevator Traffic Engineering

The HARint Plane, a Methodology for Systematic Elevator Traffic Design

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ABSTRACT

The design of an elevator system heavily relies on the calculation of the round trip time under up peak (incoming) traffic conditions. The round trip time can either be calculated analytically or by the use of Monte Carlo simulation. However, the calculation of the round trip time is only part of the design methodology.

This article presents a step-by-step automated design methodology which gives the optimum number of elevators in very specific, constrained arrival situations. A range of situations can be considered and a judgment can be made as to what is the best cost-performance trade-off. It uses the round trip value calculated by the use of other tools to automatically arrive at an optimal elevator design for a building. It employs rules and graphical methods.

The methodology starts from the user requirements in the form of three parameters: the target interval; the expected passenger arrival rate (AR%) which is the passenger arrival in the busiest five minutes expressed as a percentage of the building population; and the total building population. Using these requirements, the expected number of passengers boarding an elevator car is calculated. Then the round trip time is calculated (using other tools) and the optimum number of elevators is calculated. Further iterations are carried out to refine the actual number of passengers boarding the elevator and the actual achieved target. The optimal car capacity is then calculated based on the final expected passengers boarding the car. The HARint plane is presented as a graphical tool that allows the designer to visualize the solution.

Three different rated speeds are suggested and used in order to explore the possibility of reducing the number of elevator cars. Moreover, the average passenger travelling time is used to indicate the need for zoning of buildings.

Keywords: Elevator; lift; round trip time; interval; up peak traffic; rule base; Monte Carlo simulation; average travelling time; HARint plane; average waiting time.
Nomenclature

- $a$: the rated acceleration in m/s$^2$
- $AR\%$: the passenger arrivals expressed as a percentage of the building population arriving in the busiest five minutes
- $CC$: the rated car capacity in persons
- $CL\%$: the percentage car loading
- $d_f$: the height of a floor in m
- $H$: the highest reversal floor in a round trip (where floors are numbered 0, 1, 2,...,$N$)
- $HC\%$: the handling capacity expressed as a percentage of the building population that can be transported in five minutes
- $HC_{i}$: the initial value of the handling capacity at the start of the iteration
- $HC_{f}$: the final value of the handling capacity at the end of the iteration
- $int_{tar}$: the target value of the interval in s
- $int_{act_i}$: the initial value of actual interval at the start of the iteration in s
- $int_{act_f}$: the final value of actual interval at the end of the iteration in s
- $j$: the rated jerk in m/s$^3$
- $\lambda$: the passenger arrival rate in passengers per second
- $L$: the number of the elevators in the group
- $N$: the total number of floors above the main entrance
- $P$: the number of passengers transported in one round trip
- $P_{5\text{ min}}$: the number of passengers arriving for service in five minutes
- $P_{act_i}$: the initial value of actual number of passengers at the start of the iteration
- $P_{act_f}$: the final value of actual number of passengers at the end of the iteration
- $\Delta P_{\text{min}}$: the smallest change in the value of $P$ that will cause the iterations to terminate
- $S$: the probable number of stops in a round trip
- $\tau$: the round trip time in s
- $\tau_i$: the initial value of the round trip time at the start of the iteration in s
- $\tau_f$: the final value of the round trip time at the end of the iteration in s
- $t_{dc}$: the door closing time in s
- $t_{do}$: the door opening time in s
- $t_f$: the time taken to complete a one floor journey in s
- $t_p$: the passenger boarding time in s
- $t_{po}$: the passenger alighting time in s
- $tsd$: the motor start delay in s
- $tao$: the door advance opening time in s (where the door starts opening before the car comes to a complete standstill)
- $tv$: the time taken to traverse a floor at rated speed in s (equal to $\frac{d_f}{v}$)
- $v$: the rated speed in m/s

1. INTRODUCTION

The round trip time is the time needed by the elevator to complete a full journey in the building, taking passengers from the main entrance(s) and delivering them
to their destinations and then expressing back to the main entrance, under up peak (incoming) traffic conditions.

However, having calculated the round trip time, an automated optimal calculation methodology that guides the designer to find a suitable optimum design is required. Most methods rely on trial and error, and might not lead to an optimal design. The car capacities are usually oversized; the handling capacity might be in excess of the arrival rate.

Parts I, II, III, IV, V, VI and VII of this series of articles ([1], [2], [3], [4], [5], [6], and [7] respectively) have contributed directly or indirectly to the elevator traffic system design process. Specifically, article VI [6] presented an introductory set of rules that can be used for the design of elevator traffic systems. In contrast, this article presents a step-by-step automated methodology for elevator design under specific arrival conditions, assuming that a method exists for calculating the round trip time. It uses a combination of rules and graphical methods to arrive at an optimal solution. It allows the designer to find the optimum number of cars, the optimum elevator speed and the minimal car capacity. It also presents the designer with an objective criterion as to when to split the building into zones based on the average travelling time of the passengers. A graphical tool, called the HARint plane, is presented as a means of visualising the solution. Having obtained an answer for the specific arrival condition, the designer can then examine other arrival conditions and carry out a cost-performance tradeoff.

The fact that the methodology is fully automated makes it very attractive for implementation as a software package. It has also been used for teaching elevator traffic analysis to final year undergraduate mechatronic engineering students. More details about the HARint plane methodology can be found in [8] and [9].

Commentary on the methods used for calculating the round trip time is presented in section 2. The drawbacks of the existing elevator system design methods are discussed in section 3 where it shows that the results from these methods are not optimal and can be wasteful. Section 4 is the core of this article where it presents the full mathematical analysis for the method. Section 5 shows how graphical tools can visually convey the optimum solution. Section 6 provides two examples to illustrate the methodology. Section 7 presents two important points relating to the optimality of the method and the assurance of convergence to a solution. Section 8 provides another example to illustrate the use of the passenger average travelling time as a trigger for zoning. Conclusions are drawn in section 9.

2. CALCULATION OF THE ROUND TRIP TIME
The design of elevator systems is a multi-objective optimisation problem. It aims to minimise a number of parameters: the number, capacity and speed of the elevators used; the average waiting time and the average travelling time of the passengers using the elevators; the energy consumption of the elevators and the core space used. This article concentrates on minimising the number, speed and
capacity of the elevators while meeting the quality of service and quantity of service requirement as stipulated by the client.

The calculation of the round trip time is the basis for the classical design method of elevator design. This article does not address the issue of calculating the round trip or the passenger average travelling time. It assumes that these variables have been calculated by other means. The calculation of the round trip time can either be carried out by the use of analytical equations such as those outlined in [10], [11], [12], [13], [14], [15], [16], [17] and [18]; or by the use of the Monte Carlo simulation method as outlined in [19]; or the use of Markov Chain Monte Carlo (MCMC) as outlined in [20] and [21]. The calculation of the average travelling time and the average waiting time using analytical equation has been presented in [22] and [7]; and using the Monte Carlo simulation method for the average travelling time in [23].

3. THE PROBLEM WITH CONVENTIONAL DESIGN METHODS
The following is an overview of the traditional elevator design method. The round trip time is usually calculated analytically using equation (1) below assuming one single entrance and up peak traffic conditions:

\[ \tau = 2 \cdot H \cdot \left( t_v + S + 1 \right) \cdot \left( t_f + t_v + t_{do} + t_{dc} + t_{sd} - t_{ao} \right) + P \left( t_{pi} + t_{po} \right) \]  

(1)

Other methods can be used, such as the Monte Carlo simulation method, to calculate the round trip time [19] or MCMC as shown in [20] and [21]. The Monte Carlo simulation offers the advantage over equation (1) in that it can provide an accurate value for the round trip even where any combination of the following special conditions (or all of them) exist:

a) Unequal floor populations.

b) Unequal floor heights.

c) Multiple entrance floors.

d) Top speed not attained in one floor journey.

It will be assumed that the designer starts with the knowledge of the following parameters that are given either by the architect or the building owner or that can be inferred from the type of occupancy (e.g., office, residential...etc.). These represent the user requirements.

a) The total building population, \( U \). If this is not given directly, it can be calculated from either net floor area or the gross floor area.

b) The expected arrival rate, \( AR\% \). This is the percentage of the building population arriving in the building during the busiest five minutes. This value depends on the type of building occupancy.
c) The target interval

A sufficient design meets the following two conditions:

\[ HC\% \geq AR\% \]  \hspace{1cm} (2) \\
\[ int_{act} \leq int_{tar} \]  \hspace{1cm} (3)

A design that meets equations (2) and (3) is an acceptable design, but might not be an optimum design (i.e., it could be a wasteful design). The optimum design is one that meets the two equations shown below (4) and (5).

\[ HC\% = AR\% \]  \hspace{1cm} (4) \\
\[ int_{act} = int_{tar} \]  \hspace{1cm} (5)

In practice however, it is nearly impossible to find a design that meets both of equations (4) and (5) above. This is due the fact that the number of cars in the group, \( L \), cannot be a fraction (it has to be a whole number). Hence, in practice, an optimum solution will satisfy the two equations shown below:

\[ HC\% = AR\% \]  \hspace{1cm} (6) \\
\[ int_{act} < int_{tar} \]  \hspace{1cm} (7)

This section illustrates the main problem with the conventional design method. It relies on the user picking a suitable speed, \( v \), and a suitable car capacity, \( CC \). The user then assumes that the cars will fill up to the 80% of the car capacity.

The round trip time is then calculated based on the selected speed and the selected car capacity using equation (1) or any other suitable means. This provides a value for the round trip time, \( \tau \).

Dividing the round trip time by the target interval and rounding up the answer provides the required number elevators. The user has now two values that represent the qualitative and quantitative performance of the system: The handling capacity and the actual value of the interval, respectively. Comparing these values to the desired values, results in four possible cases, discussed in detail in Table 1 below.
Table 1: The four possible cases for any design.

<table>
<thead>
<tr>
<th>Quantitative Design Criterion</th>
<th>Qualitative Design Criterion</th>
<th>Case I</th>
<th>Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$HC% &lt; AR%$ &amp; $int_{act} &gt; int_{tar}$</td>
<td>Unacceptable design. Cannot be addressed by reducing the car loading. The designer will have to increase the number of elevators and repeat the analysis.</td>
<td></td>
<td>Unacceptable design. Might be addressed by increasing the car loading and using a larger car capacity if needed.</td>
</tr>
<tr>
<td>$HC% &gt; AR%$</td>
<td>$int_{act} &lt; int_{tar}$</td>
<td>Case III Unacceptable design. But it might be addressed by reducing the car loading.</td>
<td>Case IV Acceptable design, but might not be an optimum one. There may be further scope in reducing the number of elevators, reducing the rated speed or both.</td>
</tr>
</tbody>
</table>

Specifically, there are two problems with this method:

1. In the three cases where the design is unacceptable, the designer does not have a clear set of rules showing him/her how to move to an acceptable design (as defined in Case IV). It is a mixture of judgement, experience and trial and error.

2. Even where the user manages to get to an acceptable design by arriving at Case IV, he/she cannot be sure that he/she has an optimum solution, despite the fact that the design meets both qualitative and quantitative criteria. The designer will have to do further trial and error iterations to check that the design is optimum (e.g., further reduce the number of elevators, $L$ and then repeat the calculation of the round trip time). The main reason for this is that the designer starts from an arbitrary car size and assumes it fills up to 80% of its capacity rather than calculating the actual passenger arrival expected.

The next section attempts to address the drawback with this traditional methodology.

4. ANALYSIS AND DEVELOPMENT OF THE FORMULAE
The design methodology developed in this section allows the designer to arrive directly at a design that is optimum and within a fixed number of steps without the need for trial and error searches or iterations. This section develops the method and the associated formulae.
Developing a clearly defined methodology for design with a set of concrete steps offers the following advantages:

1. It allows designers to carry out the design regardless of their level of expertise, through clearly defined sets and rules.
2. It offers the opportunity to automate the design process in software.

The methodology presented here uses the following rule:

The following parameters should be minimised in an optimal design in the following order of importance (that reflect the cost of the whole installation):

a) Number of elevators.

b) Elevator speed.

c) Elevator capacity.

So where two solutions have a different number of elevators, the one with fewer elevators is selected; for solutions with the same number of elevators, the one with the lower speed is selected; for solutions with same number of elevators and the same speed, the one with the smaller car capacity is selected.

Nevertheless, it is accepted that there are situations where the order of priority above is not correct (e.g., the restricted headroom in the building might restrict the rated elevator speed and force the designer to use a larger number of elevators in order to force a lower rated speed). In such conditions the designer can alter the rule for the priorities and select the answers accordingly.

4.1 Optimising the Number of Elevators

The design process starts by finding actual number of passengers that will board the elevator in any round trip journey. In effect, this is the number of passengers that will board the elevator from the main entrance (in the case of a single entrance arrangement) or the number of passengers in the car when it leaves the highest arrival floor heading upwards to the destination floors (in case of multiple contiguous entrances). This depends on three parameters that are all known at the start of the design process and are usually provided by the client, the developer or the architect. These are the target interval, $\text{int}_{\text{tar}}$, the arrival rate, $AR\%$, and the total building population, $U$. This is shown in equation (8) below.

The number of passengers arriving in the peak five minutes can be found by multiplying the arrival rate by the total population, as shown below (the five minute period has traditionally been used as the design basis in elevator systems):

$$P_{5\min} = AR\% \cdot U$$  \hspace{1cm} (8)

The arrival rate can then be expressed in units of persons per second by dividing by 300 seconds per minute.
\[ \lambda = \left( \frac{AR\% \cdot U}{300} \right) \]  \hspace{1cm} (9)

Thus an initial estimate of the actual number of passengers that will arrive in a single interval can be found by multiplying the target interval by the arrival rate of passengers.

\[ P_{act} = (int_{tar} \cdot \lambda) \]  \hspace{1cm} (10)

The subscript \( i \) denotes the fact that is an initial estimate. It is worth noting that equation (10) is used in reference [24] as a tool to assess the actual interval at partial car loading, but not as a sizing tool.

It is too early at this stage to consider the car capacity. This can be done later when the final number of the passengers in the car has been determined.

Having arrived at an estimate for the actual number of passengers in the car, the next step is to find the corresponding round trip time. Using a classical method of calculating the round trip time or using Monte Carlo Simulation, the value of the round trip time can be found, where one or more of the four special conditions exist, then a variation of equation (1) can be used or one can resort to the Monte Carlo simulation method. The round trip time is in effect a function of the actual number of passengers if all other parameters are kept constant (such as the kinematics, number of floors, total building population, door timings, floor heights). This provides an initial value for the round trip time.

\[ \tau_i = f(P_{act}) \]  \hspace{1cm} (11)

From the calculated value of the round trip time, the required number of elevators can be calculated:

\[ L = \left\lceil \frac{\tau_i}{int_{tar}} \right\rceil \]  \hspace{1cm} (12)

It is worth noting that this act of rounding up is unavoidable as a whole number of elevators can only be selected. The resultant number of elevators, \( L \), is the nearest to the optimum as practically as possible.

Due to the process of rounding up to find the number of elevators required the actual value of the interval will be slightly lower than the target interval, and the actual value of the handling capacity, \( HC\% \), will be slightly higher than the arrival rate, \( AR\% \).

The actual value of the interval will be found by dividing the round trip time by the number of elevators, as shown below in equation (13):
The actual handling capacity can also be found by using equation (14) below:

\[ HC\%_i = \frac{300 \cdot P_{acti}}{U \cdot int_{acti}} \]  

(14)

This provides an acceptable solution that satisfies Case IV discussed in the last section. This is the optimum number of elevators required to meet the design criterion. However, it is not an optimum design in respect of the required car capacity. The next subsection examines the methodology to find the optimum car capacity.

4.2 Optimising the Car Capacity

The solution developed so far optimises the number of elevators, but does not provide the optimum car capacity. Thus such a solution is not complete, as it will not exist in practice due to the fact that the actual arrival rate, \( AR\% \), is less than the handling capacity, \( HC\%_i \), and hence fewer passengers will fill up car than \( P_{acti} \). In effect the actual number of passengers will be fewer than \( P_{acti} \). This results in a value of the interval that is lower than the actual interval, \( int_{acti} \).

So the car loading has to be gradually reduced until the handling capacity, \( HC\%_i \), is exactly equal to the arrival rate \( AR\% \). This gives the final value of the interval, \( int_{actf} \), that is less than the target interval, \( int_{tar} \), and a new value for the number of passengers that we will denote as \( P_{actf} \). This is carried out by applying the four equations (15) to (18).

\[ P_{actf} = \lambda \cdot int_{acti} \]  

(15)

This new value of \( P_{actf} \) is then used to evaluate the new value of the round trip time using either equation (1), Monte Carlo simulation or any other suitable method. This is simply shown below as the evaluation of the round trip time based on the new value of \( P_{actf} \).

\[ \tau_f = f(P_{actf}) \]  

(16)

This resultant value of the round trip time is then used to re-evaluate the final value of the interval, as shown below.

\[ int_{actf} = \frac{\tau_f}{L} \]  

(17)

This value of the interval is then set to the original value of the interval in order to carry out another iteration of equations (15) to (17):
The four equations (15) to (18) are then repeated in a loop as many times as necessary until an acceptable convergence has been achieved. For example, an acceptable convergence criterion might be set such that the iteration process is stopped if the change in $P$ is lower than a certain predetermined limit, as shown in equation (19) below. This effectively provides a tool for specifying the resolution of the final solution.

The iterative process shown above is similar to the Iterative Balance Method (IBM) introduced by Barney [11]; and the enhanced up peak mode in the software elevate [28].

From the value of $P_{act\,f}$, the required car capacity can be calculated by dividing $P_{act\,f}$ by the assumed maximum car loading factor and then rounding up to the nearest standard car capacity size. The maximum car loading factor has been assumed here to be 80%, but any other value can be used depending on the application.

\[
 CC = \left\lceil \frac{P_{act\,f}}{0.8} \right\rceil
\]

It is important to note that the rounding up function shown in equation (20) above, does not merely round up to the nearest integer; it also rounds up to the nearest car capacity preferred size (in units of passengers). Examples of standard car sizes are 8, 10, 13, 16, 21 and 26 persons.

The actual car loading can then be found by dividing the number of passengers, by the car capacity:

\[
 CL\% = \frac{P_{act\,f}}{CC}
\]

By definition the final value of the handling capacity is equal to the arrival rate, $AR\%$ (depending on the convergence criterion shown in equation (19)):

\[
 HC\%_f = AR\%
\]

And the final value of the interval, $int_{act\,f}$, is less than the initial estimate of the interval, $int_{act\,i}$, which in turn is less than the target interval, $int_{tar}$.
This final optimum solution optimises the car capacity as well as the number of elevators. It is represented by the three parameters:

1. The number of elevators, \( L \).
2. The car capacity, \( CC \).
3. The car loading, \( CL\% \).

Its performance is characterised by the two equations that set the quantitative and qualitative performance respectively, reproduced below:

\[
HC\%_f = AR\% \tag{24}
\]
\[
int_{act,f} < int_{tar} \tag{25}
\]

4.3 Optimising the Rated Speed of the Elevators

The development so far has assumed that the value of the rated speed is set. In many situations this is not the case and the designer has the freedom to find an optimum value for the rated elevator speed.

A general accepted industry standard is to find the rated speed by dividing the total travel distance by 20. This is an approximation that allows the elevator to travel the distance between terminal floors in 20 seconds. In practice the elevator will take longer to do this due to the time required for accelerating and decelerating and placing a limit on the value of the jerk. It is more accurate to use the exact equation for calculating the transition time that takes into consideration the values of the acceleration, deceleration and jerk [3] and [25], although this has not been shown here.

It is thus suggested that three initial values for the speed are found, by dividing by 20, 25 and 30 and rounding up the answer the nearest standard speed (e.g., 1.6, 2.0, 2.5, 3.15, 4.0 m/s...etc.).

\[
v_1 = \frac{d_{total}}{20} \tag{26}
\]
\[
v_2 = \frac{d_{total}}{25} \tag{27}
\]
\[
v_3 = \frac{d_{total}}{30} \tag{28}
\]

Using each one of the three suggested speeds from equations (22), (23) and (24) the full design process defined by equations (8) to (20) is repeated giving three possible solutions. It is worth noting that equation (11) should be modified in order to clearly show the round trip time is now dependent on both the actual number of passenger in the car as well as the rated speed. This is shown below in equation (29).
The three design solutions are then compared. The solution that provides the fewest number of elevators is chosen. Where two solutions require the same number of elevators, then the solution with the lower speed is selected. This rationale is based on the assumption that reducing the number of elevators results in a cost saving more than that offered by the reduction in speed.

A general block diagram of the software is shown in Figure 1. The user requirements are shown inside a ‘cloud’ where they specify the performance requirements that must be achieved. The arrival rate is extracted from these parameters, as well as the expected number of passengers to arrive in the interval. A round trip time calculator (not described in this article) is used to calculate the round time based on the building parameters and the number of passengers. This value of round trip is divided by the target interval in order to find the required number of elevators.

An iteration loop is then followed by using the actual value of interval to find a new value for the number of passengers expected to arrive during the interval. This is then used again to find a new value for the round trip time, until final convergence is achieved. From this final value of the round trip time, the final value of the interval and the final value of the passengers arriving during the interval are calculated.
Figure 1: Block diagram showing the automated optimal design methodology.
5. GRAPHICAL REPRESENTATION: THE HARINT PLANE

The methodology described in the last section can be represented in a graphical format. The aim of the graphical representation in this case is to allow the designer to understand the effect of changes on the resulting solution, and be able to assess how far it is from the optimum solution.

In order to develop the graphical representation, a plane is presented. This plane has two axes; the $x$-axis represents the interval in seconds and the $y$-axis represents the handing capacity. Each point on the plane represents a possible solution (not necessarily an acceptable or correct one). The point representing the optimum solution, can be located by the intersection of the vertical line representing $int_{tar}$ and the horizontal line representing $AR\%$, as shown in Figure 2. The plane is referred to as the HARint plane, as it contains the $HC\%$ and the $AR\%$ on the $y$-axis and the $int$ on the $x$-axis (HCARint abbreviated to HARint).

![HARint Plane Diagram](image)

Figure 2: The HARint plane.

Plotting lines of equal $L$ (number of elevators) values produces the HARint plane shown in Figure 3. Plotting lines of equal $P$ (number of passengers) produces the HARint plane shown in Figure 4. The HARint plane can be very useful in visualising a specific solution and appreciating the optimality or otherwise of suggested solutions.
Figure 3: The lines on the HARint plane that represent different values for $L$.

Figure 4: HARint plane with curves of equal $L$ and equal $P$ added and the optimum solution marked with a small circle.

Figure 5 shows the position of the hypothetical optimum solution on the HARint plane which is the intersection point of the $AR\%$ horizontal line and the $int_{tar}$ vertical line. It is hypothetical because it is not achievable in practice as it requires a fractional number of elevators, $L$. Applying the rounding up equation (12) and then applying the iterations from equations (15) to (18) moves the solution to the practical optimum solution (that lies on the $AR\%$ line and uses a whole number of elevators, $L$). This is shown in more detail in Figure 6 where the intermediate point is also shown (after applying equation (12) and before applying the iterations).
The use of the HARint plane shown in Figure 2 to Figure 6 has been useful for visualising the solution, but has not been used to actually find a solution by the use of graphical methods. It might be possible to find a graphical method of finding a solution for a problem by the using the HARint plane as a solution chart (e.g., as the Smith chart is used in radio engineering and the Nichols chart is used in control systems). However, before this can be achieved, a method of normalisation needs to be introduced in order to make the HARint plane a universal tool.
6. EXAMPLES
A number of numerical examples will be presented in this section, in order to illustrate the methodology developed in section 4.

Example 1
A vertical transportation design is required for an office building with ten floors above the main entrance, no basements and a total population of 550 persons. A target interval of 30 seconds is set by the client. An arrival rate of 12% is also given.

Other parameters are given as shown below:

- a) All floor heights are equal, at 4.5 m per floor.
- b) Rated speed is 1.6 m·s⁻¹.
- c) Rated acceleration is 1 m·s⁻².
- d) Rated jerk is given as 1 m·s⁻³.
- e) Door opening time is given as 2 s.
- f) Door closing time is given as 3 s.
- g) Start delay is given as 1 s.
- h) Advance door opening is given as 0.5 s.
- i) Passenger boarding time is given as 1.2 s
- j) Passenger alighting time is given as 1.2 s.

Applying equation (8) provides a value for the number of passengers arriving in the busiest five minutes:

\[ P_{5\text{min}} = AR\% \cdot U = 12\% \cdot 550 = 66 \text{ pass/5 min} \]  \hspace{1cm} (30)

Applying equation (9) gives the value of the passenger arrival rate in units of passenger per second:

\[ \lambda = \left( \frac{P_{5\text{min}}}{300} \right) = \frac{66}{300} = 0.22 \text{ pass/s} \]  \hspace{1cm} (31)

Using equation (10) provides an initial value for the number of passengers boarding each car:

\[ P_{act} = (int_{tar} \cdot \lambda) = 30 \cdot 0.22 = 6.6 \text{ pass} \]  \hspace{1cm} (32)

Using equation (1) and this initial value for \( P_{act} \) of 6.6 passengers provides an initial value for the round trip time:

\[ \tau_i = f(P_{act}) = f(6.6) = 115.9 \text{ s} \]  \hspace{1cm} (33)

Using equation (34) provides the optimum number of elevators:

\[ L = \left\lceil \frac{115.9}{30} \right\rceil = 4 \]  \hspace{1cm} (34)
As mentioned previously, this act of rounding up (from 3.86 to 4) is inevitable, as a whole number of elevators must be used. It is the reason why the two equations (4) and (5) that represent the ultimate optimum solution cannot be simultaneously met, and why the designer has to resort to simultaneously meeting the other two equations (6) and (7).

Once the optimum number of elevators has been determined, the next step is to determine the optimum car capacity, by iteratively finding the actual number of passenger boarding the car and the actual achieve interval.

The first step is to find the initial estimate for the interval that will be achieved in practice, based on the optimum number of elevators and the initial estimate of the round trip time. This is done using equation (13):

\[ \text{int}_{act} = \frac{\tau}{L} = \frac{115.9}{4} = 28.98 \text{s} \quad (35) \]

The interval has been given the subscript \( i \) to emphasise that it is an initial estimate. As this value is lower than the target interval, \( \text{int}_{tar} \), and considering that the arrival rate in passengers per second is still constant (\( \lambda \)), then the actual number of passengers that will in fact board each car will be lower, and an estimate for it can be found using equation (15):

\[ P_{act} = \lambda \cdot \text{int}_{act} = 0.22 \cdot 28.98 = 6.375 \text{s} \quad (36) \]

This new value of \( P_{act} \) is then used to evaluate the new value of the round trip time using either equation (1), Monte Carlo simulation or any other suitable method. This provides a new value for the round trip time:

\[ \tau_{f} = f(P_{act}) = f(6.375) = 114.2 \text{s} \quad (37) \]

Using equation (17) a new estimate of the actual interval (given a subscript \( f \) to denote final) can found:

\[ \text{int}_{act} = \frac{\tau_{f}}{L} = \frac{114.2}{4} = 28.55 \text{s} \quad (38) \]

This value of the interval is then set to the original value of the interval in order to carry out another iteration of equations (36), (37) and (38):

\[ \text{int}_{act} = \text{int}_{act} = 28.55 \text{s} \quad (39) \]

Following this first iteration, a number of iterations can now be carried out using equations (36) to (39), until convergence is deemed to have taken place in the value of \( P_{act} \). Applying four more iterations, the results are shown in below.
Table 2: Iteration results for example 1.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>( P_{act} ) using equation (32)</th>
<th>( \tau ) using equation (1) or Monte Carlo simulation</th>
<th>( int_{act} ) using equation (34)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second iteration</td>
<td>6.281</td>
<td>113.4</td>
<td>28.35</td>
</tr>
<tr>
<td>Third iteration</td>
<td>6.237</td>
<td>113.1</td>
<td>28.275</td>
</tr>
<tr>
<td>Fourth iteration</td>
<td>6.22</td>
<td>112.95</td>
<td>28.225</td>
</tr>
<tr>
<td>Fifth iteration</td>
<td>6.21</td>
<td>112.87</td>
<td>28.218</td>
</tr>
</tbody>
</table>

It can be seen that the change in \( P_{act} \) in the last iteration was around 0.01 which is deemed sufficient for convergence.

Using equation (20), the optimum car carrying capacity (taking into consideration the preferred value) can be calculated as follows:

\[
CC = \left[ \frac{6.21}{0.8} \right]
\]  

(40)

Where the car carrying capacity of 8 is preferred value and corresponds to a preferred load of 630 kg. The actual car loading can then be found using equation (21):

\[
CL\% = \frac{P_{act}}{CC} = \frac{6.21}{8} = 77.6\%
\]

(41)

This result provides an optimum solution for the selected speed of 1.6 m/s, as shown in Table 3 (described by the four parameters: number of cars, car carrying capacity, car loading and top speed). The table also shows the resultant round trip time and the resultant interval:

Table 3: Optimum design solution for Example 1 at a speed of 1.6 m/s.

<table>
<thead>
<tr>
<th>( v ) (m·s(^{-1}))</th>
<th>( L )</th>
<th>CC (persons)</th>
<th>CL%</th>
<th>( P_{act} ) (persons)</th>
<th>( \tau ) (s)</th>
<th>( int_{act} ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>4</td>
<td>8</td>
<td>77.6%</td>
<td>6.21</td>
<td>112.87</td>
<td>28.218</td>
</tr>
</tbody>
</table>

The fact that the actual number of passengers is not a whole number is not a problem, as it represents an average value over the consecutive round trip journeys. As expected the final handling capacity is nearly equal to the arrival rate (AR\%), as verified by equation (42):

\[
HC\% = \frac{300 \cdot \frac{P_{act}}{int_{act}}}{U} = \frac{300 \cdot 6.21}{550 \cdot 28.218} = 12.004\% \approx 12\%
\]

(42)

The solution based on a speed of 1.6 m/s is shown graphically on the HARint plane in Figure 7 below, where the final optimum solution is shown by the small circle.
Figure 7: HARint plane showing solution of Example 1 (with a speed of 1.6 m/s).

However, the solution shown in Table 3 is only applicable to a speed of 1.6 m·s\(^{-1}\). It is possible that using a speed of 2 m·s\(^{-1}\) or a speed of 2.5 m·s\(^{-1}\) could result in a more economical solution by reducing the number of elevators. The complete design process is then carried out by applying equations (8) to (21) to the two cases with speeds of 2 m·s\(^{-1}\) and 2.5 m·s\(^{-1}\).

With these higher speeds it is no longer possible to use equation (1) as the top speed will not be attained in a one floor journey. Monte Carlo simulation will be used in this case to produce a solution.

### Table 4: Optimum design solution for Example 1 for all three speeds.

<table>
<thead>
<tr>
<th>(v) (m·s(^{-1}))</th>
<th>(L)</th>
<th>CC (persons)</th>
<th>(CL%)</th>
<th>(P_{act}) (persons)</th>
<th>(\tau) (s)</th>
<th>(int_{act}) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>4</td>
<td>8</td>
<td>77.6%</td>
<td>6.21</td>
<td>112.87</td>
<td>28.218</td>
</tr>
<tr>
<td>2.0</td>
<td>4</td>
<td>8</td>
<td>68.1%</td>
<td>5.45</td>
<td>99.8</td>
<td>24.7</td>
</tr>
<tr>
<td>2.5</td>
<td>4</td>
<td>8</td>
<td>61.0%</td>
<td>4.862</td>
<td>88.3</td>
<td>22.1</td>
</tr>
</tbody>
</table>

Examination of Table 4 reveals that the speed of 1.6 m/s results in the most economical solution as the solutions using the other two speeds have not resulted in a reduction in the number of elevators. Thus the optimum solution is the one that uses four elevators running at a speed of 1.6 m/s and rated at a car carrying capacity of 8 persons (630 kg). The car loading in this case will be 77.6%.

### Example 2

Example 1 will be repeated with a larger population of 650 persons. The three speeds used are the same as the total travel distance has not changed. The results for the three speeds are shown in Table 5 below.

### Table 5: Optimum design solution for Example 2 for all three speeds.

<table>
<thead>
<tr>
<th>(v) (m·s(^{-1}))</th>
<th>(L)</th>
<th>CC (persons)</th>
<th>(CL%)</th>
<th>(P_{act}) (persons)</th>
<th>(\tau) (s)</th>
<th>(int_{act}) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>5</td>
<td>8</td>
<td>70.7%</td>
<td>5.655</td>
<td>108.84</td>
<td>21.75</td>
</tr>
<tr>
<td>2.0</td>
<td>4</td>
<td>10</td>
<td>70.4%</td>
<td>7.439</td>
<td>114.57</td>
<td>28.61</td>
</tr>
<tr>
<td>2.5</td>
<td>4</td>
<td>10</td>
<td>67.8%</td>
<td>6.779</td>
<td>104.21</td>
<td>26.07</td>
</tr>
</tbody>
</table>
Examination of Table 5 reveals that the use of a slightly higher speed of 2 m·s⁻¹ instead of 1.6 m·s⁻¹ has resulted in a saving of one elevator. So the optimum solution in this case employs four elevators running at a speed of 2 m·s⁻¹ with a rated car carrying capacity of 10 persons. The car loading is 70.4%.

The solution utilising 2 m/s speed is shown in a graphical format on the HARint plane in Figure 8 where the small circle represents the final solution.

Figure 8: HARint plane graphical representation of the solution of Example 2.

7. NOTES ON OPTIMALITY AND CONVERGENCE

Equations (10) and (12) are fundamental to the success of the method. Two points are in order, relating to optimality and convergence.

The first point relates to optimality. The use of equation (10) ensures that the number of elevators, \( L \), calculated in equation (12) is optimal. It will not change no matter how many iterations are carried out and no matter what value for \( P \text{act} \) is used.

The second points relates to convergence to compliant or non-compliant solutions. The rounding up that is applied to the number of elevators \( L \) in equation (12) is essential for convergence to a compliant solution. It ensures that the final value of the actual interval converges to its correct value following a sufficient number of iterations. In the first iteration, the actual number of passengers reduces, and this leads to a reduction in the value of the round trip time, that in turn leads to a reduction in the value of the actual interval. This in turn leads to a reduction in the actual number of passengers…and so on. Eventually the iterations converge and no further changes take place.
On the other hand, if rounding down had been applied (or has the designer been forced to select a smaller number of elevators than is required), then this will lead to a non-compliant solution. As iterations are applied, the value of the round trip time will increase, leading to an increase in the interval, which leads in turn to an increase in the actual number of passengers. This process continues and finally converges to a point on the horizontal AR% line. The interval would be non-compliant.

In effect whatever number of elevators are chosen, the iterations would lead to solution that gravitates towards the HC%=$AR%$ horizontal, as shown in Figure 9.

This can be also clearly seen on the HARint plane, where rounding down leads to an $L$ line that is outside the solution zone and no matter how many passengers board the car, the line can never pass through the solution zone. Rounding up however, gives an $L$ line that passes through the solution zone.

In general, the iterations will converge to a compliant solution if the following condition applies:

$$ L > \frac{\tau_i}{\text{int}_{\text{tar}}} $$  \hfill (43)
...and will converge to a non-compliant solution if the following condition applies:

\[ L < \frac{\tau_i}{\text{int}_{\text{sur}}} \]  

(44)

...where \( \tau_i \) is the initial result for the round trip time found from equation (11).

8. TRIGGERS FOR ZONING

It is well known that buildings exceeding 20 floors high are usually zoned. However, there is no clear objective criterion that can be used as a trigger, although some guidelines are provided in [26] based on simulation. In this article, the following rule is presented as a trigger for zoning a building. The rule states that a building must be zoned if any of the following conditions exists:

Number of required elevators exceeds 8

OR

The average travelling time exceeds 90 seconds

The average travelling time can either be calculated using analytical methods (such as [22]) or Monte Carlo simulation methods [23]. Example 3 illustrates this rule.

Example 3

The elevator system design needs to be carried out for the building the parameters of which are shown below.

User requirements:

a. Passenger arrival rate (AR\%) is 12%.
b. Total building population of 1300 persons.
c. Target interval of 30 s.

d. Number of floors above ground is 40 floors.
e. Floor height is 4.5 m (finished floor level to finished floor level).
f. One single main entrance and no basements.
g. Equal floor populations are assumed.

Kinematics:

h. Rated acceleration, \( a \), is 1.5 m·s\(^{-2}\).
i. Rated jerk, \( j \), is 1.5 m·s\(^{-3}\).

Passenger data:

j. Passenger transfer time out of the car is 1.2 s.
k. Passenger transfer time into the car is 1.2 s.

Elevator data:

l. Door opening time is 2 s.
m. Door closing time is 3 s.

n. Start delay is 1 s.

o. Advanced door opening is 0.5 s.

Preferred car speeds are: 1, 1.25, 1.6, 2.0, 2.5, 3.1, 4.0 m/s.
Preferred car capacities are: 630, 800, 1000, 1250, 1600, 2000 kg.

**Solution**
Using the automated design methodology gives the following solution:

- Round trip time: 213.7 s
- Interval: 26.7 s
- Number of elevators: 8 elevators
- Average travelling time: 97.6 s
- Handling capacity: 12%
- Optimum speed: 8 m/s
- Actual number of passengers: 13.9 passengers
- Optimum car capacity: 21 persons
- Car loading: 66.2%

However, although the number of required elevators is less than nine elevators, the average travelling time is 97.6 s which is more than the imposed limit of 90 s. This is excessive and should trigger zoning of the building.

*When zoning the building, a general rule of thumb is to split the building population in the ratio of 57% for the lower zone and 43% for the upper zone to make up for the extra travel requirement for the upper and zone and attempt to equalise the interval and the number of elevator for both zone. This rule of thumb is used for deciding on the zoning cut-off point.*

Applying this rule would split the building into two zones: 23 floors for the lower zone and 17 floors for the upper zone (ratio of 57%:43%). This gives the following solution for the two zones:

**Lower zone**
- Round trip time: 147 s
- Interval: 29.4 s
- Number of elevators: 5 elevators
- Average travelling time: 64 s
- Handling capacity: 12%
- Optimum speed: 4 m/s
- Actual number of passengers: 9 passengers
- Optimum car capacity: 13 persons
- Car loading: 69.2%

**Upper zone**
- Round trip time: 138.5 s
- Interval: 27.7 s
- Number of elevators: 5 elevators
- Average travelling time: 66 s
Handling capacity: 12%
Optimum speed: 6.3 m/s
Actual number of passengers: 6.63 passengers
Optimum car capacity: 10 persons
Car loading: 66.3%

It can be clearly seen that the interval is approximately equal between the two zones; and the number of elevators is equal. It is preferable to have an equal number of elevators between the two zones for architectural and structural reasons.

9. ADVANTAGE OVER CONVENTIONAL METHODS

It is worth highlighting at this point the main difference between this automated method and the conventional methods. Most of the conventional methods rely on the designer selecting a car capacity \( (CC) \). The designer then finds the number of passengers by multiplying the car capacity by 80% (or any other figures that he/she deems appropriate). This provides the value of \( P \) and he/she then proceeds to find the value of the round trip time using a round trip time calculator tool. The result is then divided by the target interval to find required number of elevators. Although this guarantees that the quality of service is met, it does not guarantee that the quantity of service is met. The user then finds the handling capacity and compares it to the arrival rate. If the handling capacity is lower than the arrival rate, he/she has to either increase the number of elevators, the car capacity or both, but there is currently no clear guidance as to which one to adjust and by how much. If the handling capacity is more than the arrival rate, the user has no way of knowing whether a better solution can be found that is less wasteful.

The proposed method in this article overcomes this problem by offering a clear step by step method to find the most suitable number of elevators in one calculation.

Attempts have also been made at using queuing theory to gain a better understanding of the behaviour of the elevator traffic system from a macroscopic view [27] which can guide the car capacity selection process.

10. CONCLUSIONS

A new methodology has been introduced that provides a set of rules and graphical methods that can be used to design elevator systems in buildings. The method optimises the number of elevators in the group of elevators for a building based on the user requirements of arrival rate \( (AR\%) \), target interval \( (int_{tar}) \) and the total building population \( (U) \). The methodology then optimises the speed of the elevators and then the elevator car capacity. The method allows the user to work backwards from the actual arrival rate in the building in order to find the optimum number of elevators, instead of the trial and error method.

The methodology then presents a rule that can be used for the decision on zoning of the building where the number of required elevators exceeds eight elevators or where the passenger average travelling time exceeds 90 seconds.

The methodology assumes that a method exists for accurately calculating the round trip time and the passenger average travelling time. Analytical, Monte Carlo simulation methods and Markov Chain Monte Carlo (MCMC) can be used to calculate these parameters.

Due to the automated and rule based nature of the methodology, it is very attractive for implementation in a software tool for the design of elevator traffic systems.
The method has also been successfully used in teaching the principles of elevator traffic analysis to final undergraduate mechatronic engineering students at The University of Jordan. One of the main reasons for its success is that students do not possess any past experience in elevator traffic design and hence rely on the rule base and graphical methods in reaching a compliant design.

REFERENCES


