Linearising the Model of a Tank with a pump and an orifice
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For a tank with an output orifice, assume turbulent flow.

\[ q_i(t) - q_o(t) = \frac{dV(t)}{dt} \]

\[ V(t) = h(t) \cdot A \]

where

- \( q_i(t) \) is the input flow rate
- \( q_o(t) \) is the output flow rate
- \( V(t) \) is the volume of the fluid in the tank.
- \( h(t) \) is the height of the fluid in the tank.

For turbulent flow:

\[ q_o(t) = C_d \cdot a \sqrt{2g h(t)} \]

where

- \( C_d \) is the discharge coefficient
- \( g \) is the acceleration due to gravity
- \( a \) is the area of the orifice
\[
\frac{dq_0(t)}{dh(t)} = \frac{d}{dh(t)} \left( Cd \cdot a \sqrt{2g \cdot h(t)} \right) \\
= Cd \cdot a \sqrt{2g} \left( \frac{1}{2 \sqrt{h(t)}} \right) \\
= Cd \cdot a \sqrt{\frac{g}{2h(t)}}
\]

at the quiescent point:

\[
\frac{dq_0(t)}{dh(t)} \bigg|_{h(t) = h_0} = Cd \cdot a \sqrt{\frac{g}{2h_0}}
\]

(which is a constant)

Linearizing gives:

\[
q_0(t) = \left( Cd \cdot a \sqrt{\frac{g}{2h_0}} \right) h(t)
\]

let \( R = Cd \cdot a \sqrt{\frac{g}{2h_0}} \)

\[
\Rightarrow q_0(t) = \frac{h(t)}{R}
\]

Also let us denote \( A \) as \( C \).

(2)
\[ \dot{q}_i(t) - q_0(t) = \frac{d}{dt} V(t) = \frac{d}{dt} (A \cdot h(t)) \]

\[ h(t) = q_i(t) - h(t) \]

We can think of \( q_i(t) \) as an input and \( h(t) \) as a state.

\[
\dot{h}(t) = \left( -\frac{1}{RC} \right) h(t) + \frac{q_i(t)}{C}
\]

where

\[
R = \frac{1}{Cd \cdot a \sqrt{\frac{g}{2 \kappa_0}}}
\]

\[
C = A
\]

**Typical values of the discharge coefficient**

\[ Cd : \]

- Plate: 0.61
- Orifice, sharp edges: 0.61
- Venturi nozzle, machined: 0.995