The Amended A and B Matrices in Regulators and Servos

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In a regulator with pole placement:

\[ u = -kx \quad \text{where if } u \text{ is scalar} \]

Then

- \( k \) is a \( 1 \times n \) vector
- \( x \) is a \( n \times 1 \) vector

\[ x = Ax + Bu = Ax + Kx = (A - BK)x \]

So \( \text{new } A = A - BK \)

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In a servo, let \( y = Cx = [1 \ 0 \ 0 \ 0] x \uparrow \)

\( \Rightarrow y = x_1 \)

operating with \( x \), in servo mode

\[ u = (r_1 - x_1)k_1 - (k_2x_2 + K_3x_3 - \cdots - K_nx_n) \]

\[ = r_1k_1 - x_1k_1 - (k_2x_2 + K_3x_3 - \cdots - K_nx_n) \]

\[ = r_1k_1 - (k_1x_1 + k_2x_2 + \cdots + K_nx_n) \]

\[ = r_1k_1 - kx \uparrow \text{vector} \quad \text{vector} \quad n \times 1 \]
substituting in the main equation:

\[ x = Ax - Bu = Ax + B(Kr_i - Kx) \]

\[ = Ax + BKr_i - BKx \]

\[ = Ax - BKx + BKr_i \]

\[ = (A - BK)x + (BK_i)r_i \]

Let the new input be \( r_i \)

\[ \Rightarrow \boxed{B_{\text{new}} = BK_i} \]

and \[ A_{\text{new}} = A - BK \] \( \blacksquare \)