Bernoulli's Equation applied to Hydraulic Systems

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Applying the principle of conservation of energy to the figure shown below:

\[ mgh_1 + \frac{1}{2}mv_1^2 + P_1V_1 = mgh_2 + \frac{1}{2}mv_2^2 + P_2V_2 \]

where:
- \( m \) is the mass in [kg]
- \( g \) is the acceleration due to gravity in [m/s^2]
- \( h_1 \) is the height of the first pipe in [m]
- \( h_2 \) is the height of the second pipe in [m]
- \( P_1 \) is the pressure of the fluid in the first pipe in [Pa]
- \( P_2 \) is the pressure of the fluid in the second pipe in [Pa]
- \( v_1 \) is the speed of the fluid in the first pipe in [m/s]
- \( v_2 \) is the speed of the fluid in the second pipe in [m/s]
$V_1, V_2$ are the volumes of the fluid in the first and second pipes respectively, in $[m^3]$

Assume density is $\rho_1$ and $\rho_2$ $[kg \cdot m^{-3}]$

$$\rho_1 V_1 h_1 g + \frac{1}{2} \rho_1 V_1 v_1^2 + p_1 V_1 = \rho_2 V_2 h_2 g + \frac{1}{2} \rho_2 V_2 v_2^2 + p_2 V_2$$

assuming the fluid is incompressible

$$\Rightarrow V_1 = V_2, \quad \rho_1 = \rho_2 = \rho$$

$$\rho_1 h_1 g + \frac{1}{2} \rho_1 v_1^2 + p_1 = \rho_2 h_2 g + \frac{1}{2} \rho_2 v_2^2 + p_2$$

This is expressed in pressure. We can also express it in units of length by dividing by $p_0 g$:

$$h_1 + \frac{1}{2} \frac{v_1^2}{g} + \frac{p_1}{p_0 g} = h_2 + \frac{1}{2} \frac{v_2^2}{g} + \frac{p_2}{p_0 g}$$

Elevation head

Pressure head
If a pump, a hydraulic motor and pipe frictional losses are added to the equation, it be comes:

\[ h_t + \frac{1}{2} \frac{v_1^2}{g} + \frac{P_1}{p_g} + H_p - H_m - H_f = h_z + \frac{1}{2} \frac{v_2^2}{g} + \frac{P_2}{p_g} \]

*Total head at first position*

*Total head at second position*

where:

- \( H_p \) is the head added by the pump in \([m]\)
- \( H_m \) is the head taken up by the hydraulic motor in \([m]\)
- \( H_f \) is the head taken up by frictional losses in \([m]\)