Solved Problem on Volumetric Efficiency
Example 5-1 from Exposition 7e.

A gear pump has a 3 inch outside diameter, a 2 inch inside diameter, and 1 inch width. If the actual pump flow at 1800 rpm and rated pressure is 28 gallons per minute, what is the volumetric efficiency?

\[
D_o = (3)(\frac{2.54}{1000}) = 0.0762 \text{ m}
\]
\[
D_c = 2 \times (\frac{2.54}{1000}) = 0.0508 \text{ m}
\]
\[
L = \frac{25.4}{1000} = 0.0254 \text{ m}
\]

1 gallon = 3.785 L
1 gpm = 1 gpa \times \left(\frac{\text{3.785 L}}{1 \text{ gallon}}\right) \left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) \left(\frac{1 \text{ m}^3}{60 \text{ s}}\right)
= 6.3 \times 10^{-5} \left[\text{m}^3 \cdot \text{s}^{-1}\right]

28 gpm = (28) \left(6.3 \times 10^{-5}\right) = 1.766 \times 10^{-3} \left[\text{m}^3 \cdot \text{s}^{-1}\right]

Theoretical flow rate is
\[
Q = \left[\frac{\pi}{4} \left(D_o^2 - D_c^2\right)\right] L \cdot \pi
\]
\[
= \left(\frac{\pi}{4}\right) \left(0.0762^2 - 0.0508^2\right) \times (0.0254) \left(\frac{1800}{60}\right)
\]
\[
= 1.91233 \times 10^{-3} \left[\text{m}^3 \cdot \text{s}^{-1}\right]
\]

vol. efficiency = \[
\frac{1.766}{1.91233} = 92.3\%
\]
Example 5-2:

A gear pump has a 75 mm outside, a 50 mm inside diameter, and 25 mm width. Volumetric efficiency is 90% at rated pressure. The pump speed is 1000 rpm. What is the actual flow rate at this speed?

\[
\eta_{\text{theor}} = \frac{\pi}{4} \left( \left( \frac{75}{1000} \right)^2 - \left( \frac{50}{1000} \right)^2 \right) \cdot \left( \frac{25}{1000} \right) \left( \frac{1000}{60} \right)
\]

\[= 1.02265 \left[ \text{m}^3 \cdot \text{s}^{-1} \right] \]

\[\eta_{\text{act}} = (1.02265)(0.9) = 9.204 \times 10^{-4} \text{ m}^3 \cdot \text{s}^{-1} \]

or 55.223 L/min